Please check the examination details below before entering your candidate information

Candidate surname   Other names

**Pearson Edexcel
Level 3 GCE**

Centre Number   Candidate Number

Monday 11 May 2020

Afternoon (Time: 1 hour 40 minutes)   Paper Reference **8FM0/01**

Further Mathematics

**Advanced Subsidiary
Paper 1: Core Pure Mathematics**

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations.
Calculators must not have the facility for symbolic algebra
manipulation, differentiation and integration, or have retrievable
mathematical formulae stored in them.**

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name,
  centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are
  clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers
  without working may not gain full credit.
- Inexact answers should be given to three significant figures unless
  otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P 6 2 6 8 5 A 0 1 2 8

Pearson

1. A system of three equations is defined by

$$kx + 3y - z = 3$$
$$3x - y + z = -k$$
$$-16x - ky - kz = k$$

where k is a positive constant.

Given that there is no unique solution to all three equations,

(a) show that $k = 2$

(2)

Using $k = 2$

(b) determine whether the three equations are consistent, justifying your answer.

(3)

(c) Interpret the answer to part (b) geometrically.

(1)

1.(a) For simultaneous matrix equations:

$$MX = A$$

CONSISTENT SOLUTIONS:
└ this means there are infinite solutions

if det M = 0 → no unique solution

INCONSISTENT SOLUTIONS:
└ means there is no solution

otherwise if det M ≠ 0 → there is a unique solution

$$\begin{bmatrix} K & 3 & -1 \\ 3 & -1 & 1 \\ -16 & -K & -K \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -K \\ K \end{bmatrix}$$

Given that there's no solution, det M = 0

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad \det A = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\det \begin{bmatrix} K & 3 & -1 \\ 3 & -1 & 1 \\ -16 & -K & -K \end{bmatrix} = K\begin{vmatrix} -1 & 1 \\ -K & -K \end{vmatrix} - 3\begin{vmatrix} 3 & 1 \\ -16 & -K \end{vmatrix} - \begin{vmatrix} 3 & -1 \\ -16 & -K \end{vmatrix}$$

$$= K(K+K) - 3(-3K+16) - (-3K-16)$$

$$= 2K^2 + 12K - 32 = 0$$

Question 1 continued

$K^2 + 6K - 16 = 0$ $(K+8)(K-2) = 0$

$K = \cancel{-8} \cup 2$

↳ given that K is positive
∴ reject -8

$K = 2$

b) $2x + 3y - z = 3$ ①

$3x - y + z = -2$ ②

$-16x - 2y - 2z = 2$ ③

Eliminate z by adding ① & ②

$\begin{array}{r} 2x + 3y - z = 3 \\ + \quad 3x - y + z = -2 \\ \hline 5x + 2y \qquad = 1 \end{array}$

Eliminate z by adding 2② & ③

$\begin{array}{r} 2(3x - y + z) = 2(-2) \\ + \quad -16x - 2y - 2z = 2 \\ \hline -10x - 4y \qquad = -2 \end{array}$

$-10x - 4y + 2 = -2(5x + 2y - 1)$

∴ because the 2 equations are linear multiples, the equations are consistent

c) The 3 planes form a sheaf

(i.e. infinite solutions)
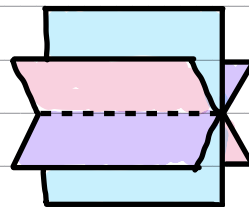
(Total for Question 1 is 6 marks)

P 6 2 6 8 5 A 0 3 2 8

2. Given that

$$z_1 = 2 + 3i$$
$$|z_1 z_2| = 39\sqrt{2}$$
$$\arg(z_1 z_2) = \frac{\pi}{4}$$

where z_1 and z_2 are complex numbers,
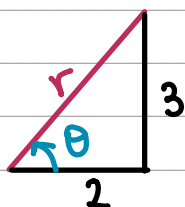
(a) write z_1 in the form $r(\cos\theta + i\sin\theta)$

Give the exact value of r and give the value of θ in radians to 4 significant figures.

(2)

(b) Find z_2 giving your answer in the form $a + ib$ where a and b are integers.
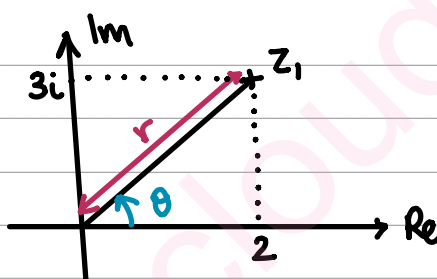
(6)

2.a) $z_1 = 2 + 3i$

$\tan\theta = \dfrac{opp.}{adj} = \dfrac{3}{2}$

$\theta = 0.9828$ rad

$r = \sqrt{2^2 + 3^2}$

$r = \sqrt{13}$

$\therefore\ z_1 = \sqrt{13}\left(\cos(0.9828) + i\sin(0.9828)\right)$

b) $|z_1 z_2| = |z_1| \times |z_2|$

$\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

$z_1 = \sqrt{13}\left(\cos(0.9828) + i\sin(0.9828)\right)$

$|z_1| = \sqrt{13}$ $\qquad \arg(z_1) = 0.9828$

if $|z_1 z_2| = 39\sqrt{2}$

$|z_2| = \dfrac{39\sqrt{2}}{|z_1|} = \dfrac{39\sqrt{2}}{\sqrt{13}} = 3\sqrt{26}$
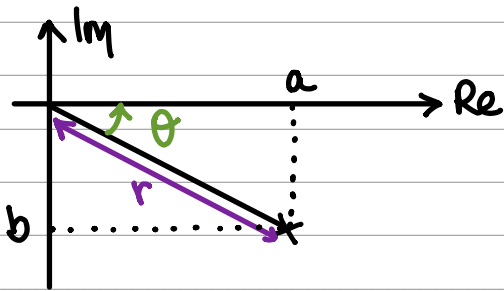
Question 2 continued

$$\text{if} \quad \arg(z_1 z_2) = \frac{\pi}{4}$$

$$\arg(z_2) = \frac{\pi}{4} - \arg(z_1) = \frac{\pi}{4} - 0.9828$$

$$\arg(z_2) = -0.1974$$

$$a = r\cos\theta = 3\sqrt{26}\cos(-0.1974)$$
$$b = r\sin\theta = 3\sqrt{26}\sin(-0.1974)$$

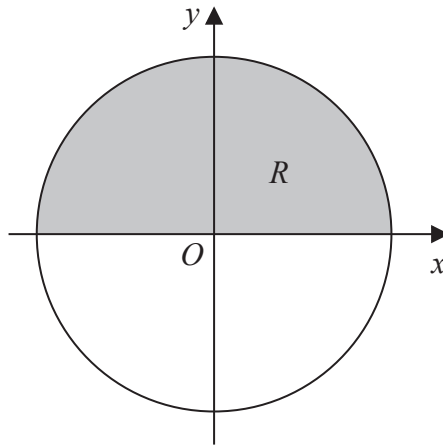$$z_2 = a + bi = 15 - 3i$$

(Total for Question 2 is 8 marks)

Turn over ▶

P 6 2 6 8 5 A 0 5 2 8

3.

Figure 1

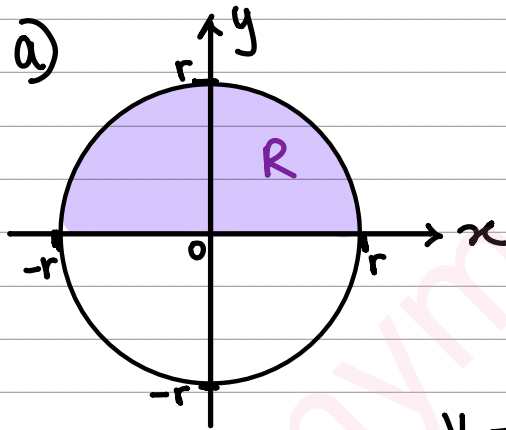Figure 1 shows a circle with radius r and centre at the origin.
The region R, shown shaded in Figure 1, is bounded by the x-axis and the part of the circle for which $y > 0$
The region R is rotated through $360°$ about the x-axis to create a sphere with volume V

Use integration to show that $V = \dfrac{4}{3}\pi r^3$

(5)

3. a)

Equation of circle:

$$x^2 + y^2 = r^2$$

$$\text{Vol}_{\substack{\text{about} \\ x\text{-axis}}} = \pi \int_a^b y^2 \, dx$$

$$V = \pi \int_{-r}^r y^2 \, dx = \pi \int_{-r}^r r^2 - x^2 \, dx$$

$$= \pi \left[r^2 x - \frac{x^3}{3} \right]_{-r}^r$$

$$= \pi \left(r^3 - \frac{r^3}{3} - \left(-r^3 + \frac{r^3}{3} \right) \right)$$

$$= \frac{4\pi r^3}{3}$$

4.   **All units in this question are in metres.**

A lawn is modelled as a plane that contains the points $L(-2, -3, -1)$, $M(6, -2, 0)$ and $N(2, 0, 0)$, relative to a fixed origin O.

(a) Determine a vector equation of the plane that models the lawn, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$

(3)

(b) (i) Show that, according to the model, the lawn is perpendicular to the vector $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

   (ii) Hence determine a Cartesian equation of the plane that models the lawn.

(4)

There are two posts set in the lawn.
There is a washing line between the two posts.
The washing line is modelled as a straight line through points at the top of each post with coordinates $P(-10, 8, 2)$ and $Q(6, 4, 3)$.

(c) Determine a vector equation of the line that models the washing line.

(2)

(d) State a limitation of one of the models.

(1)

The point $R(2, 5, 2.75)$ lies on the washing line.

(e) Determine, according to the model, the shortest distance from the point R to the lawn, giving your answer to the nearest cm.

(2)

Given that the shortest distance from the point R to the lawn is actually $1.5\,\text{m}$,

(f) use your answer to part (e) to evaluate the model, explaining your reasoning.

(1)

4.(a) $\vec{LM} = \vec{OM} - \vec{OL} = \begin{pmatrix} 6 \\ -2 \\ 0 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 8 \\ 1 \\ 1 \end{pmatrix}$

$\vec{LN} = \vec{ON} - \vec{OL} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} - \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$

— position vector

Equation of plane : $r = a + \lambda b + \mu c$

— 2 direction vectors

Question 4 continued

$$r = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 8 \\ 1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$$

b) (i) **if vectors a, b are perpendicular $a \cdot b = 0$**

∴ if the plane is perpendicular to $\begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

then both direction vectors in the plane are perpendicular

$$\begin{pmatrix} 8 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix} = 8(1) + (2) + (-10) = 0$$

$$\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix} = 4(1) + 3(2) + (-10) = 0 \quad \therefore \text{ perpendicular}$$

(ii) **cartesian equation of plane :** $\quad r \cdot n = a \cdot n$

normal to plane

point on plane

$$r \cdot \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix} = 2 \qquad \Rightarrow \qquad x + 2y - 10z = 2$$

c) $\vec{PQ} = \vec{OQ} - \vec{OP} = \begin{pmatrix} 6 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} -10 \\ 8 \\ 2 \end{pmatrix} = \begin{pmatrix} 16 \\ -4 \\ 1 \end{pmatrix}$

$$\therefore \quad r : \begin{pmatrix} -10 \\ 8 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 16 \\ -4 \\ 1 \end{pmatrix}$$

P 6 2 6 8 5 A 0 9 2 8

Question 4 continued

d) The washing line will not be straight

The lawn will not be flat

e) shortest distance $= \dfrac{|a \cdot n + d|}{|n|}$ ← $ax + by + cz + d = 0$
from a point to a plane

$r = \begin{pmatrix} 2 \\ 5 \\ 2.75 \end{pmatrix}$ $\qquad n = \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix}$

$$\dfrac{\left| \begin{pmatrix} 2 \\ 5 \\ 2.75 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix} \right|}{\left| \begin{pmatrix} 1 \\ 2 \\ -10 \end{pmatrix} \right|} = \dfrac{|2(1) + 5(2) + 2.75(-10) - 2|}{\sqrt{1^2 + 2^2 + (-10)^2}} = \dfrac{17.5}{\sqrt{105}}$$

$$= 1.71 \text{ m}$$

f) 1.7 m is significantly different to 1.5 m
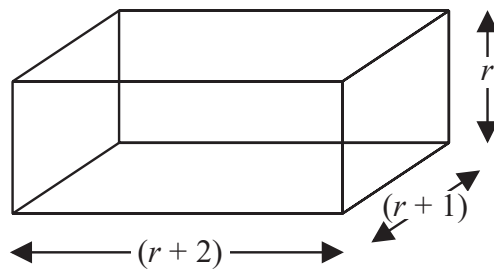
∴ the model is not a very good model

5.

Figure 2

A block has length $(r + 2)$ cm, width $(r + 1)$ cm and height r cm, as shown in Figure 2.

In a set of n such blocks, the first block has a height of 1 cm, the second block has a height of 2 cm, the third block has a height of 3 cm and so on.

(a) Use the standard results for $\sum_{r=1}^{n} r^3, \sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ to show that the **total** volume, V, of

all n blocks in the set is given by

$$V = \frac{n}{4}(n + 1)(n + 2)(n + 3) \qquad n \geqslant 1$$

(5)

Given that the total volume of all n blocks is

$$(n^4 + 6n^3 - 11\,710) \text{ cm}^3$$

(b) determine how many blocks make up the set.

(2)

5. a) Volume $= r(r+1)(r+2)$

$$= r(r^2 + 3r + 2) = r^3 + 3r^2 + 2r$$

The total volume of n blocks:

$$\sum_{r=1}^{n} (r^3 + 3r^2 + 2r)$$

$$= \sum_{r=1}^{n} r^3 + 3\sum_{r=1}^{n} r^2 + 2\sum_{r=1}^{n} r$$

USING STANDARD SUMMATIONS

$$\sum_{r=1}^{n} r^3 = \frac{1}{4} n^2(n+1)^2 \qquad \sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1) \qquad \sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

$\sqrt{}$ triangular no.s

from formula booklet

Question 5 continued

$$= \frac{1}{4}n^2(n+1)^2 + 3\left(\frac{1}{6}n(n+1)(2n+1)\right) + 2\left(\frac{1}{2}n(n+1)\right)$$

$$= \frac{n(n+1)}{4}\left(n^2 + n + 4n + 2 + 4\right)$$

$$= \frac{n(n+1)}{4}\left(n^2 + 5n + 6\right) = \frac{1}{4}n(n+1)(n+2)(n+3)$$

b) $\frac{1}{4}n(n+1)(n+2)(n+3) = n^4 + 6n^3 - 11710$

$(n^2+n)(n^2+5n+6) = 4n^4 + 24n^3 - 46840$

$n^4 + 5n^3 + 6n^2 + n^3 + 5n^2 + 6n = 4n^4 + 24n^3 - 46840$

$3n^4 + 18n^3 - 11n^2 - 6n - 46840 = 0$

$\therefore \quad n = 10$

6. (i)

$$A = \begin{pmatrix} 2 & a \\ a-4 & b \end{pmatrix}$$

where a and b are non-zero constants.

Given that the matrix **A** is self-inverse,

(a) determine the value of b and the possible values for a.

(5)

The matrix **A** represents a linear transformation M.

Using the smaller value of a from part (a),

(b) show that the invariant points of the linear transformation M form a line, stating the equation of this line.

(3)

(ii)

$$P = \begin{pmatrix} p & 2p \\ -1 & 3p \end{pmatrix}$$

where p is a positive constant.

The matrix **P** represents a linear transformation U.
The triangle T has vertices at the points with coordinates (1, 2), (3, 2) and (2, 5).
The area of the image of T under the linear transformation U is 15

(a) Determine the value of p.

(4)

The transformation V consists of a stretch scale factor 3 parallel to the x-axis with the y-axis invariant followed by a stretch scale factor –2 parallel to the y-axis with the x-axis invariant. The transformation V is represented by the matrix **Q**.

(b) Write down the matrix **Q**.

(2)

Given that U followed by V is the transformation W, which is represented by the matrix **R**,

(c) find the matrix **R**.

(2)

6.(i) a) $A \times A^{-1} = I$

∴ if $A = A^{-1}$

$\begin{bmatrix} 2 & a \\ a-4 & b \end{bmatrix} \times \begin{bmatrix} 2 & a \\ a-4 & b \end{bmatrix} = \begin{bmatrix} 2(2)+a(a-4) & 2(a)+a(b) \\ 2(a-4)+b(a-4) & a(a-4)+b(b) \end{bmatrix}$

Question 6 continued

$$= \begin{bmatrix} a^2-4a+4 & 2a+ab \\ 2a-8+ab-4b & a^2-4a+b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$a^2-4a+4=1 \qquad 2a+ab=0 \longrightarrow a(2+b)=0$$

↳ but we know

$$2a+ab-4b=8 \qquad a^2-4a+b^2=1$$

$$a \neq 0$$

$$\underline{b=-2}$$

$$a^2-4a+(-2)^2=1$$
$$a^2-4a+3=0$$
$$(a-3)(a-1)=0$$

$$\underline{a=1 \cup 3}$$

b) $$\begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$
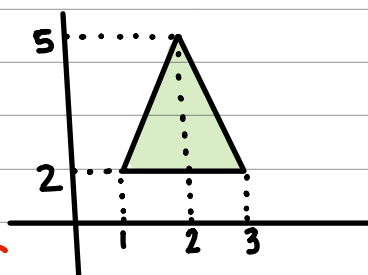
→ invariant means the same

$$\begin{bmatrix} 2x+y \\ -3x-2y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Set like components equal :

$$y=-x \qquad \therefore \quad y+x=0$$

(ii) $$P = \begin{bmatrix} p & 2p \\ -1 & 3p \end{bmatrix}$$

Area of triangle (T)

$$= \frac{1}{2} \times 2 \times 3 = 3$$

The determinant gives the scale factor for a transformation

Question 6 continued

If the area after $\cup = 15 = 5 \times 3$

$$\therefore \det(P) = 5$$

$\det(P) = 3p^2 + 2p = 5$

$$3p^2 + 2p - 5 = 0$$

$$(3p + 5)(p - 1) = 0 \qquad p = 1 \cup -\frac{5}{3}$$

but p is a positive constant

b) $\underset{\smile}{V} = $ stretch scale factor 3 parallel to x-axis followed by stretch scale factor -2 parallel to y-axis

represented by matrix Q

$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$$

Transformation A followed by B is represented by the matrix BA

$$\therefore Q = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$$

c) $W = \cup$ followed by V

$R = QP$

$$= \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 2 & -6 \end{bmatrix}$$

P 6 2 6 8 5 A 0 1 8 2 8

7.

$$f(z) = z^4 + az^3 + bz^2 + cz + d$$

where a, b, c and d are real constants.

The equation $f(z) = 0$ has complex roots z_1, z_2, z_3 and z_4
When plotted on an Argand diagram, the points representing z_1, z_2, z_3 and z_4 form the vertices of a square, with one vertex in each quadrant.
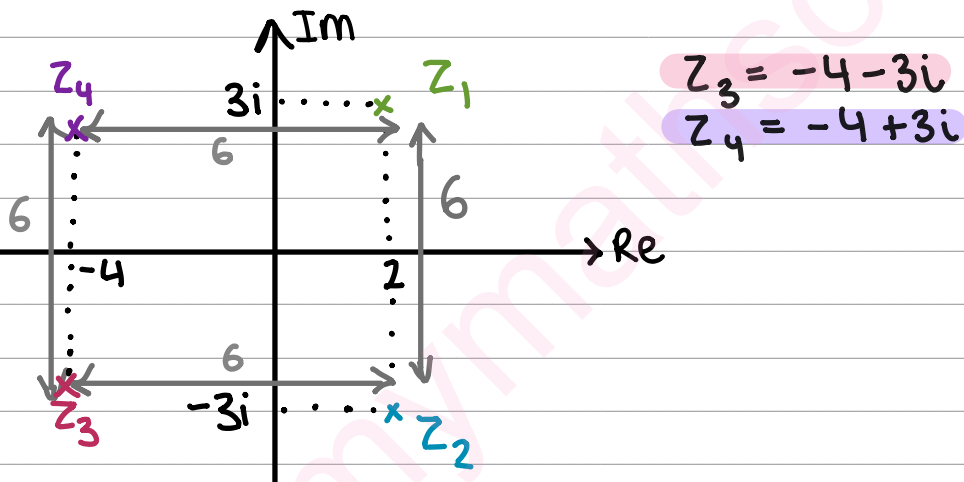Given that $z_1 = 2 + 3i$, determine the values of a, b, c and d.

(6)

7. a) $f(z) = z^4 + az^3 + bz^2 + cz + d$

Complex roots also occur in conjugate pairs
∴ if $2+3i$ is a root
$2-3i$ is a root

∴ $f(z) = (z - (2+3i))(z - (2-3i))(z - z_3)(z - z_4)$

$z_3 = -4 - 3i$
$z_4 = -4 + 3i$

$f(z) = (z^2 - 4z + 13)(z^2 + 8z + 25)$

$= (z^4 + 4z^3 + 6z^2 + 4z + 325)$

∴ $a = 4 \qquad b = 6 \qquad c = 4 \qquad d = 325$

8. Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 2^{n+2} + 3^{2n+1}$$

is divisible by 7

(6)

INDUCTION :
- Prove true for base case
- Assume true for $n=k$
- Consider $n=k+1$ & replace by assumption
- Conclusion

Base case : $n=1$

$$f(1) = 2^{(1)+2} + 3^{2(1)+1} = 2^3 + 3^3 = 35$$

$$35 = 5 \times 7 \quad \therefore \quad 7 \mid 35 \qquad \therefore \text{ statement true for } n=1$$

Assume true for $n=k$ $\qquad \therefore f(k)$ is divisible by 7

$$f(k) = 2^{k+2} + 3^{2k+1} = 7A \qquad A \in \mathbb{Z}$$

rearrange : $2^{k+2} = 7A - 3^{2k+1}$

consider $n=k+1$

$$f(k+1) = 2^{(k+1)+2} + 3^{2(k+1)+1}$$

$$= 2^{k+3} + 3^{2k+3}$$

$$= 2(2^{k+1}) + 3^{2k+3}$$

$$= 2(7A - 3^{2k+1}) + 3^{2k+3}$$

$$= 14A - 2(3^{2k+1}) + 3^2(3^{2k+1})$$

$$= 14A + 7(3^{2k+1}) = 7(2A + 3^{2k+1})$$

$$= 7B \qquad \text{where } B \in \mathbb{Z}$$

$\therefore$ hence $f(k+1)$ also divisible by 7

Question 8 continued

Since statement is true for $n=1$, and we have proved if true for $n=k$, it is true for $n=k+1$, thus by mathematical induction, the result holds true for all positive integers

(Total for Question 8 is 6 marks)

9. The cubic equation

$$3x^3 + x^2 - 4x + 1 = 0$$

has roots α, β, and γ.

Without solving the cubic equation,

(a) determine the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} + \dfrac{1}{\gamma}$

(3)

(b) find a cubic equation that has roots $\dfrac{1}{\alpha}$, $\dfrac{1}{\beta}$ and $\dfrac{1}{\gamma}$, giving your answer in the form

$x^3 + ax^2 + bx + c = 0$, where a, b and c are integers to be determined.

(3)

9. a) $3x^3 + x^2 - 4x + 1 = 0$

$$ax^3 + bx^2 + cx + d = 0$$

$$\alpha + \beta + \gamma = -\frac{b}{a} \qquad \alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a} \qquad \alpha\beta\gamma = \frac{-d}{a}$$

$$\alpha+\beta+\gamma = -\frac{b}{a} = \frac{-1}{3} \qquad \alpha\beta+\alpha\gamma+\beta\gamma = \frac{c}{a} = \frac{-4}{3} \qquad \alpha\beta\gamma = -\frac{d}{a} = \frac{-1}{3}$$

$$\frac{\alpha\beta+\alpha\gamma+\beta\gamma}{\alpha\beta\gamma} = \frac{1}{\gamma} + \frac{1}{\beta} + \frac{1}{\alpha} = \frac{-4/3}{-1/3} = 4$$

b) **METHOD 1 (using substitution):**

$$3x^3 + x^2 - 4x + 1 = 0$$

$$w = \frac{1}{x}$$

$$x = \frac{1}{w}$$

$$3\left(\frac{1}{w}\right)^3 + \left(\frac{1}{w}\right)^2 - 4\left(\frac{1}{w}\right) + 1 = 0$$

$$3 + w - 4w^2 + w^3 = 0$$

METHOD 2 (using product/sum rules):

└ using info. from part (a)

P 6 2 6 8 5 A 0 2 4 2 8

Question 9 continued

$$p \quad q \quad r$$
$$\downarrow \quad \downarrow \quad \downarrow$$

new equation with roots $\frac{1}{\alpha}$, $\frac{1}{\beta}$, $\frac{1}{\gamma}$ → new equation
$$ax^3 + bx^2 + cx + d = 0$$

$-\frac{b}{a} = p + q + r = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = 4$ (from part (a))

$$b = -4$$

$\frac{c}{a} = pq + pr + qr = \frac{1}{\alpha\beta} + \frac{1}{\alpha\gamma} + \frac{1}{\beta\gamma}$

$= \frac{\gamma + \beta + \alpha}{\alpha\beta\gamma} = \frac{-1/3}{-1/3} = 1$ $\qquad c = 1$

$-\frac{d}{a} = pqr = \frac{1}{\alpha\beta\gamma} = \frac{1}{\alpha\beta\gamma} = \frac{1}{-1/3} = -3$

$$d = 3$$

$$w^3 - 4w^2 + w + 3 = 0$$

(Total for Question 9 is 6 marks)

10. Given that there are two distinct complex numbers z that satisfy

$$\left\{z : |z - 3 - 5i| = 2r\right\} \cap \left\{z : \arg(z - 2) = \frac{3\pi}{4}\right\}$$

determine the exact range of values for the real constant r.

(7)

AND

10. $\{z : |z - (3 + 5i)| = 2r\} \cap \{z : \arg(z - (2)) = \frac{3\pi}{4}\}$

↑ circle : centre $(3, 5)$
radius $2r$

└ argument from
$(2, 0) = \frac{3\pi}{4}$

Cartesian equation for circle:

$(x - 3)^2 + (y - 5)^2 = (2r)^2$

Cartesian equation for line:

grad. $= \frac{opp.}{adj.} = \tan\left(\frac{3\pi}{4}\right) = -1$

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substitute line into circle:

$(x - 3)^2 + (-x + 2 - 5)^2 = 4r^2$

$x^2 - 6x + 9 + x^2 + 6x + 9 = 4r^2$

$2x^2 + 18 - 4r^2 = 0$

$y - y_1 = m(x - x_1)$

$y - 0 = -1(x - 2)$

$y = -x + 2$

Given that there are 2 distinct complex no.s,

there must be 2 distinct solutions to the equation

$\therefore$ discriminant $> 0 \quad \rightarrow \quad b^2 - 4ac > 0$

$2x^2 + (18 - 4r^2) = 0$

$(0)^2 - 4(2)(18 - 4r^2) = -144 + 32r^2 > 0$
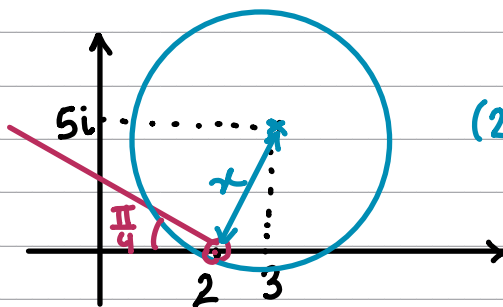
$r^2 > \frac{9}{2}$

However this ensures 2 solutions for an intersection with the circle & the line, however $\{z : \arg(z-2) = \frac{3\pi}{4}\}$ is only a half line, so any intersections below $(2,0)$ are not valid.

Drawing a diagram, we can see this occurs if the radius is bigger than a certain value

$5i$ ·······
$\frac{\pi}{4}$
2 3

if the circle touches $(2,0)$ or if x is any larger, there will only be 1 solution

$\therefore \max x = $ distance from centre $(3,5)$ to $(2,0)$

$$= \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

$$\max \text{ radius} = \max x = 2r = \sqrt{26}$$

$$\max \ r = \frac{\sqrt{26}}{2}$$

$$\therefore \text{ if } r^2 > \frac{9}{2} \quad \cap \quad r < \frac{\sqrt{26}}{2}$$

$$r < -\frac{3\sqrt{2}}{2} \quad \cup \quad r > \frac{3\sqrt{2}}{2}$$

$$\therefore \quad \frac{3\sqrt{2}}{2} < r < \frac{\sqrt{26}}{2}$$

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